

Hidden Local and Non-local Symmetries of S - matrices of $\mathcal{N} = 2, 4, 8$ SYM in $D = 2 + 1$

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This talk, based principally on [1], is devoted to properties of tree-level S -matrices of $\mathcal{N} = 2, 4, 8$ SYM in $D = 2 + 1$. We'll discuss an on-shell formalism for three-dimensional theories inspired by the spinor-helicity framework in four spacetime dimensions. Our framework will be shown to be particularly well suited for the extraction of hidden symmetries and algebraic structures that the scattering amplitudes of the three-dimensional theories possess. In particular we shall discuss the manifest $SO(\mathcal{N})$ symmetry of the S -matrix to all orders in perturbation theory; a symmetry that the Lagrangians of these theories do not have. After a brief discussion of the ramification of the $SO(\mathcal{N})$ invariance to the $D2 - M2$ brane dualities, we shall introduce an on-shell superfield framework for three-dimensional theories and end with a surprising hint of the existence of non-local symmetries for the S -matrix of the $\mathcal{N} = 8$ theory.

1. Introduction

As emphasized in various presentations in this session of the meeting, and summarized particularly succinctly in Marcus Spradlin's talk, there has been an enormous amount of progress in our understanding of S -matrices of Yang-Mills theories with high degrees of supersymmetry, particularly in four spacetime dimensions. For recent reviews of the subject, see [2]. In particular we have learned that the gauge invariant S -matrices reveal many additional symmetries and algebraic structures (Yangian invariance and dual superconformal invariance in the case of four-dimensional theories to name a few) which are difficult, if not impossible, to fathom using the standard Lagrangian formulations of the corresponding gauge theories. It is perhaps fair to say, that the wealth of insights uncovered by much of the recent work on S -matrices seems ostensibly to depend on the underlying conformal invariance that the four-dimensional theories possess, at least at tree level. Thus a natural question to ask is how the lessons learned in $D = 4$ generalize when one gives up on conformal symmetries. Yang-Mills theories in three spacetime dimensions seem to provide a fertile test bed to probe this question. In $D = 3$, one has control over the amount of supersymmetry that one might want to impose on the gauge theory, however, the fact that the Yang-Mills coupling constant is dimension-full in $D = 3$ implies that conformal symmetry is lost from the get-go. Apart from providing a controlled departure from conformal invariance, SYM theories in $D = 2 + 1$ are of intrinsic interest from the point of view of the gauge gravity duality, in particular from the point of view of applying $D2$ and $M2$ brane systems to the study of condensed matter physics. With all these motivations in mind, we'll focus on the studies of S matrices of the dimensional reductions of $\mathcal{N} = 1$ SYM from $D = 4, 6$, and 10 to $D = 3$.

We'll start with a pedagogical introduction to a spinor-helicity-like on-shell formalism and use it to study the tree-level four particle amplitudes of the S -matrix elements of the theories mentioned above in a unified way. The point of this discussion will be to show that the four particle amplitudes mirror the algebraic structures of the corresponding quantities in four-dimensional theories in a close way. Furthermore, the S -matrix will be shown to have full $SO(\mathcal{N})$ invariance, even though the Lagrangian of the corresponding theories only have manifest $SO(\mathcal{N} - 1)$ symmetry. Afterwards, we'll recast our computations in an efficient form using an on-shell superspace prescription and comment on possible implications of the enhanced symmetry of the S -matrix to the connection between $D2$ and $M2$ brane worldvolume theories. At the end of the talk, we'll construct a non-local operator, reminiscent of a level-1 Yangian generator in four dimensions, that commutes with the analogs of the MHV super-amplitudes in $D = 3$, even in the absence of dual-conformal and dual-superconformal symmetries.

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2. $SO(\mathcal{N})$ Covariant On-shell Framework

We begin with a unified description of the Lagrangians of the theories under consideration; $\mathcal{N} = 2, 4, 8$ SYM in $D = 2 + 1$, which are dimensional reductions of $\mathcal{N} = 1$ SYM theories from 4, 6, 10 dimensions respectively to three spacetime dimensions,

$$S = \frac{1}{g^2} \text{Tr} \int d^3x \left(-\frac{1}{2} F_{\bar{M}\bar{N}} F^{\bar{M}\bar{N}} + i \bar{\lambda}_A \gamma^\mu D_\mu \lambda_A + \rho_{AB}^i \bar{\lambda}_A [\Phi_i, \lambda_B] \right),$$

where $F_{\bar{M}\bar{N}}$ is the field strength in 4, 6 or 10 dimensions reduced to $D = 2 + 1$. \bar{M}, \bar{N} are the three-dimensional Lorentz indices for $\bar{M}, \bar{N} = 0, 1, 2$. For the other values of the barred indices the F^2 term is just a short-hand notation for the part of the action containing the $\mathcal{N} - 1$ scalars $\Phi^i, \{i = 2 \cdots \mathcal{N}\}$ and their interactions with the gauge field. We also have \mathcal{N} real fermions, $\lambda_A, \{A = 1 \cdots \mathcal{N}\}$. From the part of the action involving the scalars, it is easily seen that these theories have a manifestly realized $SO(\mathcal{N} - 1)$ R-symmetry at the Lagrangian level.

Keeping the charge of the talk in mind, we can already see the emergence of a putative $SO(\mathcal{N})$ structure at the Lagrangian level. The tensors (ρ) that dictate the Yukawa couplings, when combined with the obvious $SO(\mathcal{N})$ invariant, namely, the delta function satisfy the following equations:

$$\rho_{AB}^C = \{\rho_{AB}^1 = \delta_{AB}, \rho_{AB}^i\}, \quad \rho_{AC}^D \rho_{BC}^E + \rho_{AC}^E \rho_{BC}^D = 2\delta^{DE} \delta_{AB}. \quad (1)$$

These tensors have natural $SO(\mathcal{N})$ covariance properties. For $\mathcal{N} = 2$,

$$\rho_{AB}^C = \{\delta_{AB}, \epsilon_{AB}\} \quad (2)$$

are the two $SO(2)$ invariants. For $\mathcal{N} = 8$, ρ_{BC}^A are the well known $\mathbf{8}_{s,c,v}$ symbols relating the three eight dimensional representations of $SO(8)$. As we shall see, the ρ tensors are also the natural structure constants that appear in the on-shell supersymmetry algebra. To see that, we carry out an oscillator decomposition of the dynamical fields as follows

$$\begin{aligned} \Phi^i &= \int \frac{d^2p}{(2\pi)^2} \frac{1}{\sqrt{2p^0}} (a^{i\dagger}(p) e^{ip \cdot x} + a^i(p) e^{-ip \cdot x}), \\ A_\mu &= \int \frac{d^2p}{(2\pi)^2} \frac{1}{\sqrt{2p^0}} \epsilon_\mu(p, k) (a_1^\dagger(p) e^{ip \cdot x} + a_1(p) e^{-ip \cdot x}), \\ \lambda_I &= \int \frac{d^2p}{(2\pi)^2} \frac{1}{\sqrt{2p^0}} (u(p) \lambda_I^\dagger(p) e^{ip \cdot x} + u(p) \lambda_I(p) e^{-ip \cdot x}). \end{aligned} \quad (3)$$

Taking a cue from the spinor-helicity basis employed in four dimensions, we construct the polarization vector from the solutions of the massless Dirac equation as

$$\epsilon_\mu(p, k) = \frac{\langle p | \gamma_\mu | k \rangle}{\langle k p \rangle}, \quad p_\mu \epsilon^\mu(p, k) = k_\mu \epsilon^\mu(p, k) = 0. \quad (4)$$

It is implied that

$$|p\rangle = u(p), \quad \langle p| = \bar{u}(p), \quad \langle kp\rangle = \bar{u}(k)u(p) = -\langle pk\rangle, \quad (5)$$

where $u(p)$ is a solution of the massless Dirac equation in three dimensions¹.

Now, if we were studying a free abelian theory, then we would have been able to dualize the gauge field into a scalar ($\partial_\mu \Phi_1 \sim \epsilon_{\mu\nu\rho} F^{\nu\rho}$), which would have combined with the other $\mathcal{N} - 1$ scalars to form an $SO(\mathcal{N})$ invariant combination. While we do not know how to dualize the non-abelian theory in an off-shell formalism, we can combine the on-shell scalar for the gauge field a_i with the other $\mathcal{N} - 1$ scalar oscillators a_i to form an $SO(\mathcal{N})$ vector even in the non-abelian theory: $a_N = \{a_1, a_i\}$. As explained in [1] we can take advantage of various

¹ k is a reference momentum. The arbitrariness involved in the choice of k is nothing but arbitrariness of gauge choices in disguise.

properties of the polarization vector constructed above to deduce the supersymmetry transformation laws for the on-shell states to be the following:

$$Q_A^\alpha |a_B(p)\rangle = \frac{1}{2} u^\alpha(p) \rho_{AC}^B |\lambda_C(p)\rangle, \quad Q_A^\alpha |\lambda_B(p)\rangle = -\frac{1}{2} u^\alpha(p) \rho_{AB}^C |a_C(p)\rangle. \quad (6)$$

It is immediately clear from the appearance of the $SO(\mathcal{N})$ covariant quantities ρ_{BC}^A as the structure constants that the on-shell algebra is $SO(\mathcal{N})$, as opposed to $SO(\mathcal{N}-1)$, covariant. Thus for the S matrix to commute with the supercharges, it must necessarily be $SO(\mathcal{N})$ invariant. This is a statement we will back up with some evidence later in the talk. For now let us focus on the enhanced symmetry of the on-shell framework.

2.1. Symmetry Enhancement and Helicity

The enhancement of symmetry from $SO(\mathcal{N}-1)$ to $SO(\mathcal{N})$ can be traced to $\mathcal{N}-1$ factors of $U(1)$ which relate the on-shell gluon a_1 to the oscillators corresponding to the scalar fields. Focusing on the $\mathcal{N}=2$ case for simplicity; and defining the complex combinations $\mathcal{W}_\pm = \frac{1}{\sqrt{2}}(\mathcal{W}_1 \pm i\mathcal{W}_2)$, we can express the algebra on single particle states in a $U(1)$ symmetric form as

$$\begin{aligned} Q_+^\alpha |a_+\rangle &= \frac{1}{\sqrt{2}} u^\alpha |\lambda_+\rangle, & Q_+^\alpha |\lambda_-\rangle &= -\frac{1}{\sqrt{2}} u^\alpha |a_-\rangle, \\ Q_-^\alpha |a_-\rangle &= \frac{1}{\sqrt{2}} u^\alpha |\lambda_-\rangle, & Q_-^\alpha |\lambda_+\rangle &= -\frac{1}{\sqrt{2}} u^\alpha |a_+\rangle, \\ Q_-^\alpha |a_+\rangle &= Q_+^\alpha |a_-\rangle = Q_+^\alpha |\lambda_+\rangle = Q_-^\alpha |\lambda_-\rangle = 0. \end{aligned} \quad (7)$$

The emergent $U(1)$ is nothing but what used to be the little group in $D=4$. Thus, the higher dimensional helicity degree of freedom augments itself as a continuous $U(1)$ symmetry at the level of the on-shell amplitudes upon dimensional reduction. As a matter of fact, it is shown in [1] that it is precisely the dimensional reduction of the two gluon helicity states that generates the complex combination a_\pm in $D=3$. In other words, the $U(1)$ charge carried by the one particle states plays exactly the same role that helicity does in four dimensions. For instance, using the supersymmetry algebra it may be shown that the color-ordered amplitudes with ‘partons’ with the same helicity, or with at most one $U(1)$ charge different, all vanish

$$\langle ++ \cdots ++ \rangle = \langle -- \cdots -- \rangle = \langle \pm \pm \cdots \mp \pm \pm \rangle = 0. \quad (8)$$

In a three-dimensional context, all of the above states can be shown to be 1/2 BPS states. The first non-vanishing amplitude is the one with two $U(1)$ charges different; the $D=3$ analog of the MHV amplitude. Quite remarkably it is given also by the famous Parke-Taylor formula

$$\langle a_+ a_+ \cdots a_- \cdots a_- \cdots a_+ \rangle = \frac{2\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \cdots \langle n1 \rangle}, \quad (9)$$

where i, j are the momenta corresponding to the ‘-’ excitations. Of course, the spinor products here are different than the four-dimensional ones, and are given by (5). However, the algebraic structure of the amplitudes is exactly the same as in the four-dimensional case.

2.2. Four Particle Amplitudes and $SO(\mathcal{N})$ Invariance

Let us now focus on four particle amplitudes for the $\mathcal{N}=2, 4, 8$ theories. There is only one independent four particle amplitude, which we can take to be the MHV amplitude given above. All others can be related to this amplitude by the action of the supersymmetry generators. For instance:

$$\begin{aligned} \langle \lambda_+ \lambda_- a_+ a_- \rangle &= +\frac{\langle 32 \rangle}{\langle 31 \rangle} \langle a_+ a_- a_+ a_- \rangle, & \langle \lambda_+ \lambda_- a_- a_+ \rangle &= +\frac{\langle 42 \rangle}{\langle 41 \rangle} \langle a_+ a_- a_- a_+ \rangle, \\ \langle \lambda_+ \lambda_- \lambda_- \lambda_+ \rangle &= +\frac{\langle 14 \rangle}{\langle 42 \rangle} \langle a_+ a_- \lambda_- \lambda_+ \rangle = +\frac{\langle 13 \rangle}{\langle 24 \rangle} \langle a_+ a_- a_- a_+ \rangle. \end{aligned} \quad (10)$$

All the relations between the different color-ordered four particle amplitudes of the $\mathcal{N}=2$ theory (which have also been explicitly checked by tree-level computations) are given in detail in [1]. If we write the amplitudes in

a $SO(\mathcal{N})$ symmetric form, then we can make the $SO(\mathcal{N})$ symmetry of these amplitudes manifest. To display a few classes of amplitudes:

$$\begin{aligned}\langle a_{A_1} a_{A_2} a_{A_3} a_{A_4} \rangle &= -2\delta_{A_1 A_2} \delta_{A_3 A_4} \frac{\langle 13 \rangle \langle 24 \rangle}{\langle 12 \rangle \langle 34 \rangle} + 2\delta_{A_1 A_3} \delta_{A_2 A_4} \\ &\quad + 2\delta_{A_1 A_4} \delta_{A_2 A_3} \frac{\langle 13 \rangle \langle 24 \rangle}{\langle 23 \rangle \langle 41 \rangle}\end{aligned}$$

$$\begin{aligned}\langle \lambda_{A_1} \lambda_{A_2} \lambda_{A_3} \lambda_{A_4} \rangle &= 2\delta_{A_1 A_2} \delta_{A_3 A_4} \frac{\langle 13 \rangle^2 \langle 23 \rangle}{\langle 12 \rangle^2 \langle 41 \rangle} - 2\delta_{A_1 A_3} \delta_{A_2 A_4} \frac{\langle 34 \rangle}{\langle 12 \rangle} \\ &\quad - 2\delta_{A_1 A_4} \delta_{A_2 A_3} \frac{\langle 13 \rangle^2 \langle 12 \rangle}{\langle 14 \rangle^2 \langle 34 \rangle}\end{aligned}$$

Defining $\rho^{A_1 A_2} \equiv \frac{1}{2} \left((\rho^{A_1})^T \rho^{A_2} - (\rho^{A_2})^T \rho^{A_1} \right)$,

$$\langle a_{A_1} a_{A_2} \lambda_{A_3} \lambda_{A_4} \rangle = -\delta_{A_1 A_2} \delta_{A_3 A_4} \frac{\langle 13 \rangle^2}{\langle 12 \rangle^2} \left(\frac{\langle 13 \rangle}{\langle 14 \rangle} + \frac{\langle 23 \rangle}{\langle 24 \rangle} \right) + \left(\rho^{A_1 A_2} \right)_{A_3 A_4} \frac{\langle 31 \rangle}{\langle 14 \rangle}.$$

The key point here is that all the amplitudes only involve the $SO(\mathcal{N})$ invariant delta function which substantiates the claim made earlier in the talk based on the on-shell superalgebra i.e. the S -matrices of the theories under consideration have a manifest $SO(\mathcal{N})$ symmetry, even though their Lagrangians don't².

3. D2 vs M2

One of the reasons to focus on the $SO(8)$ symmetry of the maximally supersymmetric $D = 3$ SYM theory is to understand the flow of the theory to its infrared fixed point where it is expected to be described by a superconformal Chern-Simons-Matter (SCS) theory. In the case of the $SU(2)$ gauge group the conformal theory is expected to be the BLG theory [3], which is of course a special case of the ABJM model [4]. The conformal theory has a manifestly realized $SO(8)$ R-symmetry, while the SYM theory only has $SO(7)$ symmetry. One of the outstanding issues about the flow of the SYM theory had been to understand how the enhancement of the global symmetries takes place. However, as far as on-shell quantities are concerned, we have shown that the $SO(8)$ symmetry is manifestly realized to all orders in perturbation theory. Fortified by our results so far, we can begin to compare the structures of the four particle amplitudes of the two theories.

All the four particle amplitudes of a large class of SCS theories with $\mathcal{N} \geq 4$ supersymmetry were computed in [5]. The amplitudes take on the form:

$$S^{CS}(W_I, W_J, W_K, W_L) = S^{CS}(k, s, t) S_{IJKL}(s, t) \quad (11)$$

where the universal part $S_{IJKL}(s, t)$ is determined by the supersymmetry algebra alone. i, j, k, l stand for the flavor $SO(8)$ indices in the case of the BLG theory. All the dependence on the Chern-Simons level number (the coupling constant) is contained in the function $S^{CS}(k, s, t)$, which also depends on the Mandelstam variables. Further more, as far as the Yang-Mills amplitudes are concerned, we have already seen that they are of the form:

$$S^{YM}(W_I, W_J, W_K, W_L) = S^{YM}(g^2, s, t) \tilde{S}_{IJKL}(s, t) \quad (12)$$

where the leading order term in $S^{YM}(g^2, s, t)$ is given by the Parke-Taylor formula given before. Furthermore, using the $SO(8)$ covariant form of the SUSY algebra, it is easy to show [1] that since both $S_{IJKL}(s, t)$ and $\tilde{S}_{IJKL}(s, t)$ follow from the same underlying algebra:

$$S_{IJKL}(s, t) = \tilde{S}_{IJKL}(s, t). \quad (13)$$

²We refer to [1] for the explicit $SO(\mathcal{N})$ invariant forms of the remaining amplitudes.

Thus at the level of the four particle amplitudes, the connection between $D2$ and $M2$ brane theories can be reduced to the problem of understanding the interpolating function which starts off as the Parke-Taylor formula and reduces to $S^{CS}(k, s, t)$ at infinite g^2 . Given the rapidly growing body of exciting developments regarding amplitudes of superconformal Chern-Simons theories [6], this is certainly a worthwhile direction to follow for future research.

4. All Tree-Level Amplitudes via Superfields and a Potential Yangian-Like Symmetry

Moving beyond four particle amplitudes at the tree level, it is very convenient to introduce an on-shell superfield to aid computations. The superfield relevant to the maximally supersymmetric gauge theory can be regarded as a dimensional reduction of the $D = \mathcal{N} = 4$ superfield, which is usually expressed as [7]:

$$\Phi = G^+ + \eta_a \lambda^a - \frac{1}{2!} \eta_a \eta_b S^{ab} - \frac{1}{3!} \eta_a \eta_b \eta_c \lambda^{abc} + \eta_1 \eta_2 \eta_3 \eta_4 G^- \quad (14)$$

where η_i are anti-commuting variables carrying $SU(4)$ indices. Apart from the six scalars S^{ab} one also has the two gluon polarization states, G^\pm , which from the three-dimensional sense are to be regarded as complex combinations of the on-shell gluon and the extra scalar obtained by the toroidal compactification of the fourth component of the gauge field.

One also has two $SU(2)$ indices $\alpha, \dot{\alpha}$ which are contracted according to the rules

$$\tilde{\lambda}_{\dot{\alpha}} \tilde{\omega}^{\dot{\alpha}} = \langle \tilde{\lambda} \tilde{\omega} \rangle, \quad \lambda_\alpha \omega^\alpha = [\lambda \omega]. \quad (15)$$

Dimensional reduction at the level of the superfield simply corresponds to identifying the two $SU(2)$ indices, which also leads to an identification of the two spinor products given above. For instance, The $D = 4$ supercharges are:

$$q^{\alpha a} = \lambda^\alpha \frac{\partial}{\partial \eta_a}, \quad \bar{q}_a^{\dot{\alpha}} = \tilde{\lambda}^{\dot{\alpha}} \eta_a, \quad (16)$$

which after dimensional reduction become

$$q^{\alpha a} = \lambda^\alpha \frac{\partial}{\partial \eta_a}, \quad \bar{q}_a^\alpha = \lambda^\alpha \eta_a. \quad (17)$$

With these rules of dimensional reduction in place, we can (as in four dimensions) express all the three-dimensional MHV amplitudes as

$$\mathcal{A}_n^{mhv} = \frac{\delta^3(p) \delta^8(\tilde{q})}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}, \quad \delta^8(\tilde{q}) = \frac{1}{2^4} \prod_{a=1}^4 \sum_{i,j=1}^n \langle ij \rangle \eta_{ia} \eta_{ja}. \quad (18)$$

More explicitly, all the MHV amplitudes with flipped $U(1)$ charges for the i and j entries are given by³

$$\mathcal{A}_n^{mhv}(i, j) = \frac{1}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle} \prod_{a=1}^4 \left(\langle ij \rangle + \langle ik \rangle \bar{\eta}_j^a \eta_{ka} - \langle jk \rangle \bar{\eta}_i^a \eta_{ka} - \frac{1}{2} \langle kl \rangle \bar{\eta}_i^a \bar{\eta}_j^a \eta_{ka} \eta_{la} \right) \quad (19)$$

where sum over repeated momentum indices is implied. $\bar{\eta}$ is understood to be the Fourier conjugate variable to η in the sense that for a superfield $\Phi(\eta)$, the adjoint superfield is obtained by

$$\bar{\Phi}(\bar{\eta}) = \int d\eta_1 d\eta_2 d\eta_3 d\eta_4 e^{\eta_a \bar{\eta}^a} \Phi(\eta).$$

In [1], a precise relation between the on-shell degrees of freedom of a four-dimensional gauge theory and those of the $D = 3$ SYM theory obtained by its dimensional reduction were given. We can augment those relations

³If one wants to work only with a $D = 3$ theory with non-maximal supersymmetry, then the superfield methods of [9] can be readily used in a three-dimensional context after they are subject to the same dimensional reduction mentioned above.

by the on-shell dimensional reduction recipe given above at the level of the superfields. Now, a closed on-shell formula for *all* tree-level amplitudes of $\mathcal{N} = 4$ SYM in $D = 4$ was given in [8]. The simple procedure described above now allows us to use those results to get the answers for any desired tree-level amplitude in $D = 3$ as well, albeit in a somewhat indirect form.

Finally, let us point out that using the superfield formalism one can explicitly show that the $D = 3$ tree amplitudes do *not* possess conformal or dual-conformal symmetries. Yet, the tree-level theories are as solvable as their four-dimensional counterparts (all the amplitudes as we argue above, are known) which suggests that there might be a Yangian-like non-local symmetry underlying the maximally supersymmetric theory in three dimensions as well. In the absence of dual conformal invariance, we cannot utilize the methods of [10] to probe the existence of Yangian symmetries or lack thereof in the theories of interest to us. However, taking a more pedestrian approach, we can construct a non-local charge that annihilates the most symmetric of the three-dimensional amplitudes, namely the MHV amplitudes given in (18). Let us consider the following generator:

$$\hat{P}^{\alpha\beta} = P^{\{\alpha\beta\}} \wedge \mathcal{D} + P^{\delta\{\alpha} \wedge L_{\delta}^{\beta\}} + \bar{q}_c^{\{\beta} \wedge q^{c\alpha\}}, \quad (20)$$

where $A \wedge B = \sum_{i < j} (A_i B_j - B_i A_j)$, the subscripts i and j label legs, and the curly brackets denote symmetrization of indices. \mathcal{D} and L are the dilatation and the $SU(2)$ generators respectively, realized on-shell by:

$$\mathcal{D} = \frac{1}{2} \frac{\partial}{\partial \lambda^\gamma} \lambda^\gamma, \quad L_{\beta}^{\alpha} = \lambda^{\alpha} \frac{\partial}{\partial \lambda^{\beta}} - \frac{1}{2} \delta_{\beta}^{\alpha} \lambda^{\gamma} \frac{\partial}{\partial \lambda^{\gamma}}. \quad (21)$$

A direct computation shows that

$$\hat{P}^{\alpha\beta} \mathcal{A}_n^{mhv} = 0. \quad (22)$$

The generator annihilating the amplitude is very reminiscent of the level-1 bosonic Yangian generator for the tree level S -matrix of $\mathcal{N} = 4$ SYM in $D = 4$ [10], and SCS theories in $D = 3$ [6]. This result clearly hints at the possibility of non-local symmetries in the S -matrix of three-dimensional maximally supersymmetric Yang-Mills theory. As mentioned before, the absence of dual conformal symmetries prevents us from generalizing this result to non-MHV amplitudes. Perhaps a twistorial approach [11] will help us determine if this non-local symmetry is just an artifact of the MHV states or a true hidden symmetry of the S -matrix.

5. Conclusions and Future Directions

To summarize, we have shown that amplitudes of $D = 3$ SYM theories have additional symmetries which the corresponding Lagrangians do not have in a manifest form. In particular, we have argued (and explicitly demonstrated at the level of the four particle amplitudes) that the S -matrices of $\mathcal{N} = 2, 4, 8$ SYM theories are manifestly $SO(\mathcal{N})$ invariant, even though the actions only have $SO(\mathcal{N} - 1)$ R-symmetry. Furthermore, as seen by the Parke-Taylor form of the amplitudes in $D = 3$, the explicit structures of the amplitudes have been shown to mirror many of the properties of the amplitudes of the superconformal $\mathcal{N} = 4$ SYM in $D = 4$, including a potential underlying non-local symmetry. The additional symmetries and the spinor-helicity framework presented in the talk is useful for studying the connections between $D2$ and $M2$ brane theories on-shell as well.

We would like to think of these results as an invitation to further explore properties of S -matrices of $D = 3$ SYM theories. We are presently studying the nature of the loop-corrections and potential twistorial reformulations for the tree-level results presented here. We hope to report more results to complement what has been presented here soon.

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